# DIOPHANTINE APPROXIMATIONS, FLOWS ON HOMOGENEOUS SPACES AND COUNTING

Abstract. Let $\xi$ be a real irrational and let $\psi : [1, \infty) \to (0, \infty)$ be a monotone increasing function. We consider the set

$$E_\xi(\psi, Q) = \{(p, q) \in \mathbf{Z}^2; |p + q\xi| < \frac{\psi(Q)}{Q}, 0 < q < Q\},$$

and we study its cardinality $|E_\xi(\psi, Q)|$ as $Q$ becomes large. In this talk we present the following result: if $|p/q + \xi| \gg q^{-\beta}$ for all $(p, q) \in \mathbf{Z} \times \mathbf{N}$ then

$$|E_\xi(\psi, Q)| \sim 2\psi(Q) \quad \text{as } Q \to \infty$$

for every monotone increasing function $\psi : [1, \infty) \to (0, \infty)$ with $\psi(Q)/Q^{1-\frac{1}{\beta-1}} \to \infty$. The exponent $1 - \frac{1}{\beta-1}$ is sharp. A similar result holds in the inhomogeneous case and a slightly weaker result if one restricts the counting to coprime $p, q$. We also discuss some of the ingredients such as a certain flow on a homogeneous space of lattices.