RECURRENCE SEQUENCES IN POSITIVE CHARACTERISTIC

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Suppose that $u(n)$, $n \in \mathbb{Z}$ is a recurrence sequence with values in a field K, and consider the set S of all integers n for which $u(n) = 0$. If K has characteristic 0, then the Skolem-Mahler-Lech theorem states that S is the union of a finite set and finitely many arithmetic progressions, but it is difficult (if not impossible) to determine the set S effectively. If K has possitive characteristic, then the structure of S may be more complicated, but one can effectively compute S (assuming one can effectively compute in K). More generally, suppose that $u_1(n), \ldots, u_m(n)$ are recurrence sequences and consider the set

$$S = \{(n_1, \ldots, n_m) \in \mathbb{Z}^m \mid u_1(n_1) + \cdots + u_m(n_m) = 0\}.$$

If K has characteristic 0, then S is not decidable (confirming a conjecture of Cerlienco, Mignotte, and Piras). But this set is effectively computable if the characteristic is positive. This is joint work with David Masser.