Local behaviour in the average of some infinite series

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(joint work with Michel Balazard)

Let $X = \mathbb{R} \setminus \mathbb{Q}$ and consider the Gauss map

$$\begin{aligned} T : X &\to X \\ x &\mapsto \{1/x\}, \end{aligned}$$

where $\{\cdot\}$ denotes the fractional part, and $(T^k)_{k\geq 0}$ its iterates. We characterize the Lebesgue points of the L^1-function

$$W(x) = \sum_{n\geq 1} (-1)^n x T(x) \dots T^{n-1}(x) \log(1/T^n(x)).$$

This series, introduced by Wilton in 1933, has a direct link with the Davenport series $\sum \frac{B(nx)}{n}$ where B denotes the first normalized Bernoulli function. This work enables us to study the differentiability of the function

$$A(x) = \int_0^\infty \{xt\}\{t\} \frac{dt}{t^2}$$

introduced in 2004 by Báez-Duarte, Balazard, Landreau and Saias in the context of the study of the Nyman criterion for the Riemann Hypothesis.